Electromagnetic waves and photons

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Abstract

We explore how the thermal ground states of two mixing and pure SU(2) Yang-Mills theories, $\mathrm{SU(2)_{CMB}}$ of scale $\Lambda_{\mathrm{CMB}} \sim 10^{-4}\,\mathrm{eV}$ and $\mathrm{SU(2)}_e$ of scale $\Lambda_e \sim 5 \times 10^5\,\mathrm{eV}$, associate either wave or particle aspects to electromagnetic disturbances during thermalisation towards the photon gas of a blackbody, in realising the photoelectric effect, and through the frequency dependence of the monochromatic, nonthermal beam structure in Thomson/Compton scattering.

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In deriving the field equations of classical electrodynamics Maxwell assumed that electromagnetic disturbances are those of a medium – the luminiferous aether – required for them to propagate in analogy to distortions of the stationary flow of a fluid in hydrodynamics [1]. If existent then such a medium needs to be of a peculiar nature, however, since it fails to associate with a preferred rest frame, a feat first demonstrated by Michelson and Morley [2] and re-confirmed many times ever since: the speed of light c is a constant of nature and as such does not depend on the observer's state of motion relative to a source. Consequences of this experimental fact, expressed by the group of Lorentz transformations linking inertial frames, are laid out by Special Relativity [3] and have been vindicated by countless experiments. In classical electrodynamics, constancy of the phase velocity c of electromagnetic waves is implied by the constancy of ϵ_0 and μ_0 – the electric permittivity and magnetic permability of free space. On the other hand, experience associates a particle-like or quantum nature to the carrier of the electromagnetic force – the photon [4, 5, 6] – whose energy $E = 2\pi\hbar\nu$ and momentum modulus $p = \frac{2\pi}{c}\nu$ are independent of the intensity of the monochromatic electromagnetic wave it associates with, but proportional to frequency ν. Here $\hbar$ denotes the (reduced) fundamental quantum of action. From now on we use natural units $c = \hbar = k_B = \epsilon_0 = \mu_0 = 1$, k_B indicating Boltzmann's constant.

The purpose of this note is to propose a framework, based on the extension of the gauge group of electromagnetism U(1) to a product of mixing SU(2) groups belonging to pure Yang-Mills theories. As we will argue, such a setting promises to reconcile the seemingly paradoxical wave-particle aspects of electromagnetic disturbances in terms of a nontrivial vacuum structure. In addressing the latter, the key instrument is the thermal ground state of SU(2) Yang-Mills theory. This concept is rooted in topologically nontrivial field configurations (trivial-holonomy Harrington-Shepard calorons and anticalorons [7] of topological charge-modulus unity) of period $\beta = 1/T$ in the Euclidean time coordinate τ, T being a temperature parameter, and in a spatial coarse graining [8]. It is important to note that the derivation of the thermal ground state does not require the consideration of thermal excitations. Therefore, it should be generalisable to the description of isolated excitation modes of the effective gauge field such that parameter T enjoys a nonthermal interpretation.

Effectively, that is, after spatial coarse graining, the thermal ground state is characterised by an inert, adjoint scalar field ϕ and an effective, pure-gauge configuration a_μ^{bg}, the latter solving the Yang-Mills equations subject to a source term provided by the former [16]. While field ϕ represents spatially densely packed caloron/anticaloron centers – their dependence on Euclidean time τ coarse grained into a mere choice of gauge at the spatial scale $|\phi|^{-1}$ [8] – the effective gauge field a_μ^{bg} represents the collective effect of caloron/anticaloron overlap, accompanied by transient holonomy changes [9, 10, 11, 12], as facilitated by their static peripheries. Caloron/anticaloron peripheries set in at spatial scale $s = \pi \frac{|\phi|^{-2}}{\beta}$ [13, 14, 15] where

$$s(\lambda) = \frac{1}{2}\lambda^2\Lambda^{-1}, \quad (\lambda \equiv 2\pi T/\Lambda), \tag{1}$$

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and Λ denotes the Yang-Mills scale of the SU(2) theory. This introduces a finite energy density $\rho^{\mathrm{gs}} = 4\pi\Lambda^3 T$ to the thermal ground state [16] which, due to the (anti)caloron's (anti)selfduality, would vanish in isolation [17].

For a given caloron, the peripheral field strength is that of a selfdual, static dipole[1] [14]. On the other hand, field ϕ breaks the SU(2) gauge symmetry of the underlying, classical Euclidean Yang-Mills action down to U(1) which means that only one of the three directions of the SU(2) algebra su(2) is massless, the (large) mass of the other two directions being fixed by (low-temperature) ambient thermodynamics in a large bulk volume [16, 18, 19]. A natural question to ask is under what conditions the associated electric and magnetic dipole densities of the thermal ground state can be regarded a medium induced by, at the same time supporting, wave-like propagation of the massless mode. By promoting a_μ^{bg} (in unitary gauge: $\phi = 2|\phi|t^3$, $|\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$, generators t^a ($a = 1, 2, 3$) normalised as $\mathrm{tr}\, t^a t^b = \frac{1}{2}\delta^{ab}$) to a monochromatic electromagnetic wave $a_\mu^{a=3}$, associated with the massless su(2) direction[2], and by identifying its mean Minkowskian energy density with ρ^{gs}, one arrives at [15]

$$|\mathbf{E}_e| = \Lambda^2 \sqrt{2\lambda}\,, \tag{2}$$

where $|\mathbf{E}_e|$ represents the mean electric field-strength modulus of $a_\mu^{a=3}$. Thus parameter $T = \frac{\lambda\Lambda}{2\pi}$ is set by the wave's intensity. Using this relation and exploiting that the dipole density is given by the ratio of dipole moment per (anti)caloron to the spatial coarse-graining volume, it was shown in [15] that the electric permittivity ϵ_0 and the magnetic susceptibility μ_0 of the thermal ground state are independent of T.

Eq. (2) together with the condition that wavelength l must not resolve the interior of a caloron/anticaloron, $l \gg s(\lambda)$, imply the following T independent statement [15]

$$|\mathbf{E}_e|^4 l^{-1} = |\mathbf{E}_e|^4 \nu \ll 8\Lambda^9\,, \tag{3}$$

Λ thus determines the maximum of intensity at a given frequency ν and vice versa commensurate with wave-like propagation. Although derived from the two T dependent relations $l \gg s(\lambda)$, see (1), and (2) condition (3) should be regarded universal. That is, in a nonthermal situation, ν is not required to satisfy any additional

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[1]The case of a small caloron scale parameter $\rho = |\phi|^{-1} \ll \beta$ was also discussed in [14]. Here the static and selfdual dipole emerges for spatial distances $r \gg \beta$ and for $\lambda \ll (2\pi)^{2/3}$. Such a situation is, however, inconsistent with the derivation of the thermal ground state [8, 16].

[2]In SU(2)$_{\mathrm{CMB}}$ massive modes $a_\mu^{a=1,2}$ interact with the massless mode $a_\mu^{a=3}$ by tiny radiative corrections only at a thermodynamical temperature bounded from below by that of the present CMB [16] and thus can be ignored in the effective Yang-Mills equation $D^\mu G_{\mu\nu} = 2ie[\phi, D_\nu\phi]$ when discussing the propagation of $a_\mu^{a=3}$. This equations thus reduces to the vacuum Maxwell equation $\partial^\mu F_{\mu\nu}^3 = 0$ with $F_{\mu\nu} = \partial_\mu a_\nu^{a=3} - \partial_\nu a_\mu^{a=3}$ which, indeed, is solved by a plane wave. The latter is subject to undetermined normalisation, frequency, and phase. Note that, as is the case for a_μ^{bg}, the Euclidean, time averaged energy density $\mathrm{tr}\frac{1}{2}(\mathbf{E}_e^2 - \mathbf{B}_e^2)$ of $a_\mu^{a=3}$ vanishes such that solely the potential $\mathrm{tr}V(\phi) = \rho^{\mathrm{gs}}$ in the effective action determines the mean energy density of such a plane wave [16].

constraint as implied by the existence of a critical thermodynamical temperature $\lambda_c = 13.87$ for the deconfining-preconfining phase transition [16].

For $\mathrm{SU}(2)_{\mathrm{CMB}}$, the Yang-Mills factor proposed to underly all experimentally investigated thermal photon gases including the Cosmic Microwave Background (CMB) [16], one has $\Lambda_{\mathrm{CMB}} \sim 10^{-4}\,\mathrm{eV}$ [18]. According to (3) the energy density $|\mathbf{E}_e|^2$ is bounded by $|\mathbf{E}_e|^2 \ll \sqrt{8 \frac{\Lambda_{\mathrm{CMB}}^9}{\nu}}$. For $\nu = 10^6\,\mathrm{Hz}$ (radio frequency) one obtains $|\mathbf{E}_e|^2 \ll 2.3 \times 10^{-21}\,\mathrm{J\,cm^{-3}}$. For a comparison, the energy density of the CMB at this frequency, measured with a spectral band width of $\Delta\nu = 10^4\,\mathrm{Hz}$, is $8\pi T \nu^2 \Delta\nu = 3.51 \times 10^{-37}\,\mathrm{J\,cm^{-3}}$. Thus, such a radio wave could represent a signal discernible from the thermal noise of the CMB. Higher-frequency monochromatic waves are bounded by energy densities reduced by a factor $1/\sqrt{\frac{\nu}{10^6\,\mathrm{Hz}}}$, and it is clear that condition (3) is violated for a wealth of phenomena, attributed to the propagation of electromagnetic waves, when setting $\Lambda = \Lambda_{\mathrm{CMB}}$.

The way out is to postulate the existence of additional SU(2) factors [16, 20] with Yang-Mills scales hierarchically larger than Λ_{CMB} which, thermodynamically seen, are in confining phases under ambient conditions such that massive modes do not propagate. One could consider $\Lambda = \Lambda_e \sim 5 \times 10^5\,\mathrm{eV} \sim m_e$, m_e denoting the electron mass. Then (3) no longer is in contradiction with experience: propagation of high-intensity and high-frequency massless waves is accomodated by the large value of the Yang-Mills scale Λ_e.

The process of thermalisation in $\mathrm{SU}(2)_{\mathrm{CMB}}$ towards blackbody radiation, which is surrounded by a cavity wall providing emitting and absorbing electrons, would then proceed as follows. At a thermodynamical wall temperature T with $\Lambda_e > T \gg T_{\mathrm{CMB}} = 2.725\,\mathrm{K} = \frac{13.87}{2\pi}\Lambda_{\mathrm{CMB}}$ [18] radiation emitted by the wall electrons satisfies (3) with $\Lambda = \Lambda_e$. This radiation represents classical waves in $\mathrm{SU}(2)_e$. A priori, their spectral energy density thus is given by the Rayleigh-Jeans law

$$u_{\mathrm{RJ}} = 8\pi T \nu^2 = \frac{2}{\pi} T^3 x^2 , \quad (x \equiv 2\pi\nu/T) , \tag{4}$$

which expresses an obvious and well-known ultraviolet catastrophe. The latter, however, does not take place if classical $\mathrm{SU}(2)_e$ waves excite *photons* from the thermal ground state of $\mathrm{SU}(2)_{\mathrm{CMB}}$. Namely, setting $\Lambda = \Lambda_{\mathrm{CMB}}$ in a thermal situation, the condition that wavelength l must be larger than s (see Eq. (1)) for wave-like propagation amounts to

$$l = \frac{2\pi}{xT} \gg s = \frac{2\pi^2 T^2}{\Lambda_{\mathrm{CMB}}^3} \quad \Leftrightarrow \quad x \ll \frac{1}{\pi}\left(\frac{\Lambda_{\mathrm{CMB}}}{T}\right)^3 . \tag{5}$$

Hence, condition (5) is violated at extremely small frequencies already, that is, for $x > \frac{1}{\pi}\left(\frac{\Lambda_{\mathrm{CMB}}}{T}\right)^3$. For such frequencies the quantum of action, localised in thus probed caloron/anticaloron centers [13, 15], participates in the thermodynamics of fluctuations by indeterministic materialisations of quanta of energy and momentum $2\pi\nu$. In assuming that their numbers are suppressed by associated Boltzmann weights the

Bose-Einstein distribution function $n_B(x) = 1/(\mathrm{e}^x - 1)$ is implied, corresponding to a spectral energy density

$$u_{\mathrm{Planck}} = \frac{2}{\pi} T^3 \frac{x^3}{\mathrm{e}^x - 1} \,. \tag{6}$$

Function u_{Planck} peaks at $x = 2.82$, is normalisable to $\frac{\pi^2}{15} T^4$ (Stefan-Boltzmann law), and is bounded from above by u_{RJ}. This provides for an energetic reason why the "rotation" from $\mathrm{SU}(2)_e$ to $\mathrm{SU}(2)_{\mathrm{CMB}}$ is invoked in the emergence of blackbody radiation. Fixing the critical, thermodynamical temperature T_c for the deconfining-preconfining phase transition in $\mathrm{SU}(2)_{\mathrm{CMB}}$ as $T_c = T_0 = 2.725\,\mathrm{K}$ [18], one obtains $\frac{1}{\pi}\left(\frac{\Lambda_{\mathrm{CMB}}}{T}\right)^3 = 1.68\,\mathrm{GHz}$. This supports the claim in [18] that the CMB radio access, see [21] and references therein, indeed is attributed to evanescent $\mathrm{SU}(2)_{\mathrm{CMB}}$ waves whose spectral energy density is forced to be maximal at $\nu = 0$ by a T-dependent Meissner mass, signalling the onset of this phase transition.

To view photons as thermal excitations of the thermal ground state in $\mathrm{SU}(2)_{\mathrm{CMB}}$ would relate to the photoelectric effect in the following way. In $\mathrm{SU}(2)_e$ an incident monochromatic wave of frequency, say, $\sim 10^{14}\,\mathrm{Hz}$, drives the dissipation of radiation-field energy within a thin surface layer of a bulk metal or semiconductor (the classical skin effect with skin depth, e.g., in copper, of a few nanometers at $\nu \sim 10^{14}\,\mathrm{Hz}$) such that an equilibrium between energy entry into this surface layer and heat flow towards the bulk is established. Such an equilibrium is characterised by a thermodynamical temperature $T \ll \Lambda_e = m_e$. By the above argument, this local thermal environment, however, is subject to $\mathrm{SU}(2)_{\mathrm{CMB}}$. Since in $\mathrm{SU}(2)_{\mathrm{CMB}}$ $s(T)$ in Eq. (1) is much larger than $l = \nu^{-1}$ the excitations of the thermal ground state are *photons*. That is, the incoming $\mathrm{SU}(2)_e$ wave of intensity $|\mathbf{E}_e|^2$ is not supported within such a thermal surface layer: by probing the interior of $\mathrm{SU}(2)_{\mathrm{CMB}}$ calorons/anticalorons it "decays" into *photons* of energy and momentum $2\pi\nu$. With a finite probability [22], such a quantum of energy and momentum is transferred to a layer electron which, in turn, is expelled from the material thus becoming a photoelectron. Modulo a material dependent work function (a function of material parameters such as skin depth, electric and heat conductivity, etc.), the maximal kinetic energy of photoelectrons thus is given by $2\pi\nu$ while their flux is proportional to the wave's intensity (energy conservation after the above described dynamical equilibrium is established).

Finally, the thermal ground state of $\mathrm{SU}(2)_e$ ($\Lambda = \Lambda_e = m_e$) could explain on a deeper level the transition from Thomson (T) scattering of a classical *wave* to Compton (C) scattering of a *photon* off an electron, which is well described by Quantum Electrodynamics. The associated total cross section [23] – an intensity independent quantity – is given as

$$
\begin{aligned}
\sigma_C &= \frac{3}{4}\sigma_T \left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \log(1+2x) \right) + \frac{1}{2x}\log(1+2x) - \frac{1+3x}{(1+2x)^2} \right] \\
&= \sigma_T[1 - 2x + \frac{26}{5}x^2 + O(x^3)], \quad (x \equiv \nu/m_e) \,.
\end{aligned} \tag{7}
$$

As x rises to order unity, an increasingly strong violation of (3) takes place [3]. Here the scattering off an isolated electron does not generate any local, thermal equilibrium but the incoming beam itself is of an increasingly corpuscular structure as x grows.

To summarise, we have discussed how condition (3) for the wave-like propagation of electromagnetic disturbances, demanding that caloron/anticaloron centers in the thermal ground state are not excited [15], and the violation of this condition may underly wave-particle duality of electromagnetic disturbances in their propagation and interaction with the stable electric charge of the electron. A single SU(2) Yang-Mills theory, conjectured to describe the thermodynamical situation – SU(2)$_{\text{CMB}}$ [16, 18] –, does not in general account for wave-like, nonthermal propagation. Therefore, a product of (at least) two mixing SU(2) Yang-Mills theories of hierarchically different Yang-Mills scales had to be postulated. Our brief conceptual discussion necessarily leaves a number of important questions unanswered. For example, is the liberation of a quantum of energy $2\pi\nu$ within a caloron/anticaloron center by an external drive field of frequency ν understandable as a resonant excitation involving monopole shaking [14, 15], which is resolved by this very liberation? Or, how can one accomodate the electroweak interactions between neutral and unstable charged leptons – very successfully and effectively described by the present Standard Model of Particle Physics – in a framework of mixing, *pure* SU(2) Yang-Mills theories. And how would this extend to incorporate the electroweak interactions of hadrons. We hope to be able to shed more light on such questions in the future.

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[3]Since σ_C is defined as an intensity independent quantity the level of violation of (3) is to be inferred at *constant* $|\mathbf{E}_e|^4$. In a nonthermal situation $\Lambda_e = m_e$ is the only mass scale in SU(2)$_e$, and thus it is natural to set $|\mathbf{E}_e|^4 = m_e^8$. As a consequence, (3) requires $\nu \ll 8\,m_e$ to obtain a beam void of particle-like aspects.

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